https://doi.org/10.62836/iaet.v3i1.232

Global Science Publishing

IAET

Innovations in Applied Engineering and Technology
https://ojs.sgsci.org/journals/iaet

Article

Finite Element Model Calibration with Surrogate Model-Based Bayesian Updating: A Case Study of Motor FEM Model

Guojun Zhang [1] and Tong Zhou [2,*]

[1] *Quadrant International Inc., San Diego, CA 92121, USA; ez900113@gmail.com*
[2] *Air China Cargo Co., Ltd., Beijing 101318, China*

Abstract: This paper introduces an advanced methodology for the calibration of Finite Element Models (FEM) utilizing a surrogate model-based Bayesian updating framework. The approach is exemplified through a case study on motor design, where precise FEM calibration is essential for predicting and optimizing motor performance. Traditional calibration techniques are often computationally expensive due to the iterative nature of the simulation process. To mitigate this, the proposed method integrates surrogate models to approximate FEM simulations, significantly reducing the computational burden without sacrificing accuracy. Bayesian updating is then employed to iteratively refine the surrogate model by incorporating new data, thereby enhancing prediction accuracy. This dual approach not only accelerates the calibration process but also ensures a high level of precision, making it highly suitable for complex engineering applications requiring both efficiency and reliability. The case study underscores the effectiveness of this methodology, demonstrating its potential to streamline the design process in motor development and other FEM-dependent engineering fields. The findings suggest that the surrogate model-based Bayesian updating approach achieves robust calibration with significantly fewer simulations, thereby optimizing both time and computational resources.

Keywords: model calibration; bayesian updating; motor design; surrogate model; optimization

1. Introduction

Finite Element Model (FEM) Calibration is a critical process in computational engineering that involves adjusting the parameters of a finite element model to ensure that its predictions accurately reflect real-world behavior. The purpose of this calibration is to reduce discrepancies between the model’s output and experimental or observed data, thereby enhancing the model’s predictive capability and reliability. At its core, FEM Calibration is an iterative process that requires the identification of uncertain parameters within the model, such as material properties, boundary conditions, and loading scenarios. These parameters are initially estimated based on theoretical knowledge or prior experiments. However, due to the inherent complexities and nonlinearities in engineering systems, these estimates often need refinement. According to the state-of-the-art research represented, the process of model calibration can be equivalent to a Bayesian Updating problem, which focuses on estimating the posterior distribution of the interested parameters in the Finite Element Model. Similar calibration methods are applied in other machine learning fields, such as optimizing fault prediction and

Received: 2 September 2024; Accepted: 22 September 2024.

* Corresponding: Tong Zhou (zhoutong@airchinacargo.com)

diagnosis in complex systems using unsupervised learning to enhance model performance and applicability in real-world scenarios [1,2].

Over the past few years, the estimation of posterior distributions has predominantly relied on Markov Chain Monte Carlo (MCMC) simulation techniques [3–5]. While other approximation-based approaches, such as the Laplace approximation method, have demonstrated commendable computational efficiency, their accuracy diminishes significantly as the number of random variables increases and the posterior density becomes more complex [6,7]. The MCMC-based Bayesian updating approach operates by generating sample points through a proposal function, with the mean value of this function dynamically adjusting according to the last accepted point. These samples are then evaluated against a standard uniformly distributed random variable to determine their acceptance or rejection. It has been observed that the probability density of the accumulated accepted points approximates the posterior probability density because the design values of each state in the Markov chain align with the prior distribution. However, a significant limitation of the MCMC-based Bayesian updating method is its failure to reliably converge to a stationary state, which is essential for the validity of the Markov chain [8–11]. This performance optimization strategy has been validated in other complex systems, such as image recommendation algorithms in social networks, where deep neural networks process large volumes of user data to enhance recommendation accuracy and efficiency [12,13]. To overcome this limitation, Ching et al. proposed the transitional Markov Chain Monte Carlo (TMCMC) simulation, which focuses on sampling from regions where probability densities are less complex. Although TMCMC enhances the performance by mitigating the burn-in phenomenon present in conventional MCMC approaches, its efficacy diminishes as the dimensionality of the parameter space increases [14–19]. To address these challenges, Straub et al. introduced an innovative methodology known as Bayesian Updating with Structural Reliability Methods (BUS). The core concept of BUS is to reformulate Bayesian updating problems into structural reliability problems. This is achieved by introducing an auxiliary standard uniform random variable and employing a simple rejection sampling strategy to select points that satisfy an equivalent limit state equation. In this context, the Bayesian updating framework for evaluating the posterior distribution is effectively treated as a component structural reliability problem with the corresponding augmented random variables, where the primary objective is to identify failure points. By circumventing the need to ensure the stationarity of the Markov Chain, BUS employs subset simulation techniques to focus solely on sampling points within the accepted domain, irrespective of the dimensionality of the random variables. This approach adaptively refines and constrains the failure domain until the targeted number of samples with the posterior distribution is achieved. As a result, BUS exhibits considerable robustness in Bayesian updating, particularly when applied within subset simulation-based techniques. A similar approach is used in robustness analysis of complex systems, employing unsupervised learning and adaptive simulation to handle high-dimensional data uncertainties [20,21].

However, there are still two major shortcomings of BUS. First, the process of estimating the posterior distribution through BUS is quite computationally expansive especially when the likelihood functions (e.g., Finite element model) becomes very complex [5,7]. It is the reason that the method for solving the structural reliability problem in BUS is through the Subset simulation, which needs to investigate a large number of evaluations to the performance function. Second, due to the failure probability estimated from the limit state function in BUS is typically very rare as the number of measurements increases, other non-simulation-based techniques such as First and Second Order Reliability Methods (FORM&SORM) become inefficient or even invalid. As pointed in [5], the computational expansive problem for BUS can be overcome through the application with surrogate model-based structural reliability methods. Those methods include, but are not limited to Response Surface [9–11], Polynomial Chaos Expansion [22], Support Vector Regression [23,24], or Kriging [25,26]. Among all of those methods, adaptive Kriging-based reliability method have been proved to be the most accurate and efficient method in solving the structural reliability problems [25,27,28], and also gained a lot of attentions and developments in recent years [29,30]. Unfortunately, the levels of failure probability (e.g., accepting ratio in BUS) in structural reliability problems and Bayesian updating problems are hugely different. As mentioned before, the failure probability in Bayesian updating can even be even smaller than 10^{-6} as the number of observations increased, which is unusual in structural reliability problems. Because of that, the number of candidate design samples in adaptive Kriging-based reliability method should be correspondingly very large to ensure a well-trained Kriging surrogate model [25]. In such a circumstance, the implementations of adaptive Kriging-based structural reliability methods in conjunction with BUS become extremely computationally inefficient. To strategically applying the adaptive-Kriging based structural reliability methods in BUS algorithm, the limit state function in BUS with very rare probability should be decomposed into several parallel limit state functions with relatively high probabilities. Similar techniques are used in optical character recognition and image denoising, where end-to-end deep learning models enhance data processing and computational efficiency [31].

Despite the advantages of Bayesian Updating with Structural Reliability Methods (BUS), it still has two significant limitations. First, the computational demands of estimating the posterior distribution through BUS can be exceptionally high, especially when dealing with complex likelihood functions, such as those involving Finite Element Models (FEM). This challenge arises because BUS relies on Subset Simulation to solve the structural reliability problem, necessitating a large number of performance function evaluations, which significantly increases computational costs. Second, the BUS approach encounters inefficiency when the failure probability, estimated from the limit state function, becomes extremely rare as the number of measurements increases. In such cases, non-simulation-based techniques, like the First and Second Order Reliability Methods (FORM & SORM), become less effective or even invalid. In other high-dimensional data tasks, distributed data parallel acceleration has been used to optimize the performance of generative adversarial networks, highlighting its advantages in handling complex datasets [32]. As highlighted in the literature, the computational burden of BUS can be mitigated by incorporating surrogate model-based structural reliability methods, such as Response Surface Methods, Polynomial Chaos Expansion, Support Vector Regression, and Kriging. Among these, adaptive Kriging-based reliability methods have emerged as particularly accurate and efficient for solving structural reliability problems and have gained considerable attention and development in recent years. However, it is crucial to note that the failure probabilities encountered in structural reliability problems differ significantly from those in Bayesian updating problems. In Bayesian updating, the failure probability (e.g., the acceptance ratio in BUS) can be exceedingly small, which is uncommon in typical structural reliability contexts. Consequently, the number of candidate design samples required for an adaptive Kriging-based reliability method must be substantially large to ensure a well-trained Kriging surrogate model. This requirement renders the implementation of adaptive Kriging-based methods within the BUS framework computationally prohibitive. To address this challenge, a strategic approach involves decomposing the limit state function in BUS, which corresponds to very rare probabilities, into several parallel limit state functions with relatively higher probabilities. This decomposition can enhance the efficiency of applying adaptive Kriging-based structural reliability methods within the BUS algorithm, allowing for more manageable and computationally feasible analyses. To address this challenge, studies suggest that improved models can be decomposed into multiple high-probability parallel paths, a method applied not only in optimizing transportation planning and routing problems but also in revealing market response patterns in finance [33 – 35]. Similarly, improved Bayesian methods in economics, like machine learning-based credit card customer segmentation, show potential for decision support and economic stability [36, 37]. It is expected that the Bayesian updating can be facilitated to improve the computational performance of machine learning [38–48].

In this paper, Bayesian updating along with formulas are briefly introduced in Section 2. Afterwards, the surrogate model-based model calibration is elaborated in Section 3. Subsequently, a numerical example of three-phase induction motor is investigated to showcase the supreme functionality of the proposed method in Section 5. Finally, conclusions are drawn in Section 6.

2. Bayesian Updating

When direct measurement of the parameters of a mechanical system is technically challenging or prohibitively expensive, engineers often rely on indirect observations to infer the status of these parameters. For example, the natural frequencies of a building can serve as an indirect indicator of its inter-story stiffness values. Generally, as the number of observations increases, the uncertainty associated with these parameters diminishes.

Bayesian updating plays a crucial role in this process by initially assuming a prior probability distribution of the parameters, denoted as $f(\mathbf{x})$, and subsequently refining this to a posterior probability distribution, $f'(\mathbf{x})$, based on the available observations. The posterior distribution is typically estimated using Bayes' theorem, which is expressed as:

$$f'(\mathbf{x}) = \frac{L(\mathbf{x})f(\mathbf{x})}{\int_{\Omega} L(\mathbf{x})f(\mathbf{x})d\mathbf{x}} \tag{1}$$

where Ω represents the probabilistic domain of the random variable $\mathbf{x}$, and $L(\mathbf{x})$ is the likelihood function, which can be defined as:

$$L(\mathbf{x}) \propto \Pr\left(Z|\mathbf{X}=\mathbf{x}\right) \tag{2}$$

In the context of Markov Chain Monte Carlo (MCMC) simulations, the denominator $\int_{\Omega} L(\mathbf{x})f(\mathbf{x})d\mathbf{x}$ in the equation is often treated as a normalizing constant and can be ignored since it merely ensures that the posterior distribution integrates to one. The likelihood function $L(\mathbf{x})$ typically comprises three components: the observed data Z, the model responses $L(\mathbf{x})$, and the error term ε, which quantifies the deviation of $h(\mathbf{x})$ from Z. Due to measurement and modeling errors, the observations Z cannot perfectly reflect $h(\mathbf{x})$, and the relationship can be expressed as:

$$\varepsilon = Z - h(\mathbf{x}) \tag{3}$$

The likelihood function $L(\mathbf{x})$ can be estimated through the probability density function (pdf) of the error ε, which is represented as:

$$L(\mathbf{x}) = \rho_{\varepsilon}(\varepsilon) = \rho_{\varepsilon}\left(Z - h(\mathbf{x})\right) \tag{4}$$

Here, ρ_{ε} denotes the pdf of ε. Although the likelihood function is often assumed to follow a multivariate Gaussian distribution with zero mean, it can also adopt other unbiased distributions. When multiple independent observations are available, the likelihood function can be expanded to:

$$L(\mathbf{x}) = \prod_{i=1}^{m} L_i(\mathbf{x}) = \prod_{i=1}^{m} \rho_{\varepsilon_i}\left(Z_i - h_i(\mathbf{x})\right) \tag{5}$$

In this discussion, the likelihood function $L(\mathbf{x})$ is used uniformly for both independent and dependent observations.

2.1. Simple Rejection Sampling (SRS)

The concept of transforming Bayesian updating into a structural reliability problem using a simple rejection sampling algorithm was first introduced by Straub and Papaioannou. This approach forms the basis of the Bayesian Updating with Structural Reliability Methods (BUS). The primary goal of Bayesian updating is to estimate the posterior distribution $f'(\mathbf{x})$, which is proportional to the product of the likelihood function $L(\mathbf{x})$ and the prior distribution $f(\mathbf{x})$:

$$f'(\mathbf{x}) \propto L(\mathbf{x})f(\mathbf{x}) \tag{6}$$

The conventional Markov Chain Monte Carlo (MCMC) methods often struggle with computational efficiency, particularly in maintaining the stability of the Markov chain. As an alternative, the simple rejection sampling algorithm can be employed. This method defines an accepted domainΩ_{acc} within the augmented outcome space $[\mathbf{x}, p]$, introducing an auxiliary random variable P:

$$\Omega_{acc} = \left[p \le cL(\mathbf{x})\right] = \left[h(\mathbf{x}, p) \le 0\right] \tag{7}$$

where $h(\mathbf{x}, p) = p - cL(\mathbf{x})$, and c is a constant such that $cL(\mathbf{x}) \le 1$ for all possible outcomes of $\mathbf{X}$. Typically, c is determined as:

$$c = \frac{1}{\max\left(L(\mathbf{x})\right)} \tag{8}$$

The posterior distribution $f'(\mathbf{x})$ can then be expressed as:

$$f'(\mathbf{x}) = \frac{\int_{p \in \Omega_{acc}} f(\mathbf{x})dp}{\int_{[\mathbf{x},p] \in \Omega_{acc}} f(\mathbf{x})dpd\mathbf{x}} = \frac{\int_0^1 I^{acc}\left([\mathbf{x},p] \in \Omega_{acc}\right) f(\mathbf{x})dp}{\int_{\mathbf{X}}\int_0^1 I^{acc}\left([\mathbf{x},p] \in \Omega_{acc}\right) f(\mathbf{x})dpd\mathbf{x}} \tag{9}$$

Here, $I^{acc}\left([\mathbf{x},p] \in \Omega_{acc}\right)$ acts as the indicator function for the structural reliability problem with the limit state function $h(\mathbf{x},p) = p - cL(\mathbf{x})$. The numerator and denominator in the final term of this equation can be further simplified:

$$\int_{p \in \Omega_{acc}} f(\mathbf{x})dp = \int_0^{cL(\mathbf{x})} f(\mathbf{x})dp = cL(\mathbf{x})f(\mathbf{x}) \tag{10}$$

and

$$\begin{aligned} &\int_{[\mathbf{x},p] \in \Omega_{acc}} f(\mathbf{x})dpd\mathbf{x} = \int_{\mathbf{X}}\int_0^1 I^{acc}\left([\mathbf{x},p] \in \Omega_{acc}\right) f(\mathbf{x})dpd\mathbf{x} = \\ &\int_{\mathbf{X}}\left\{\int_0^1 I^{acc}\left(p \le cL(\mathbf{x})\right)dp\right\} f(\mathbf{x})d\mathbf{x} = \int_{\mathbf{X}} cL(\mathbf{x})f(\mathbf{x})d\mathbf{x} \end{aligned} \tag{11}$$

This formulation aligns exactly with Bayes' theorem. While a simple rejection sampling algorithm offers a straightforward approach to Bayesian updating, it has significant limitations. Specifically, the acceptance rate diminishes substantially as the number of observations m increases. Straub and Papaioannou noted that when all measurements are independent and identically distributed (iid). This inefficiency makes the Bayesian updating process increasingly impractical, especially when dealing with complex models like the Finite Element Model, as it results in a large number of rejected samples, thereby reducing the accuracy of the posterior estimation.

2.2. The BUS Algorithm

Due to the inherent disadvantage exists in simple rejection sampling-based Bayesian updating, MCMC was proposed to overcome this limitation (e.g., low acceptance rate). However, to ensure a stable Markov chain, the MCMC-based Bayesian updating needs to investigate a large number of evaluations to the likelihood function. On the other hand, though the acceptance rate of simple rejection sampling-based Bayesian updating approach is low, it is very straightforward to implement and can guarantee an exact and uncorrelated posterior distributed sample. By maintaining those advantages in simple rejection-based approach, Straub and Papaioannou [6] proposed Bayesian Updating with Structural Reliability Methods (BUS) that strategically integrates the simple rejection sampling with structure reliability methods.

In BUS algorithm, the Bayesian updating problem is handled in a way of solving the reliability analysis problem. The equivalent limit state function in BUS approach can be read as,

$$h(\mathbf{x},p) = p - cL(\mathbf{x}) \tag{12}$$

note that the task of Bayesian updating is different from that in reliability analysis. In the process of reliability analysis, the target is to estimate the probability of failure while drawing the samples in the accepted (failure) domain is the main purpose of BUS. Concerning this point, many existing reliability analysis methods such as First & Second Order Reliability Methods (FORM&SORM), Importance Sampling (IS) and Subset Simulation (SS) should be adjusted to be applicable in association with BUS. For instance, the combination of subset simulation and BUS has shown great efficiency in drawing samples from posterior distribution. Details of BUS with subset simulation haven been shown in Algorithm 2. However, BUS algorithm has to face some challenging aspects. First, the acceptance rate tends to be extremely small when the number of observation increases significantly. In this circumstance, estimate of posterior distribution is equivalent to analysis reliability with rare events, which becomes rather computationally expansive for simulation-based approach includes subset simulation. To elaborate this point, the number of subsets can be denoted as N_{ss}, thus the total number of evaluations to the likelihood function N_{call} can be inferred as:

$$N_{call} = N_{ss} \bullet N_{in} + N_t - N_{in} \tag{13}$$

Note that N_{call} can easily exceed a thousand in BUS with subset simulation. Although this number is relative smaller compared to the crude Monte Carlo simulation, it is still computationally inefficient for Bayesian updating in sophisticated numeric models. Moreover, the limit state function $h(\mathbf{x}, p)$ can be highly non-linear since $h(\mathbf{x}, p)$ includes an integrated likelihood function $L(\mathbf{x})$, which contains the probability density function (pdf) of Gaussian distribution. This may lead to an inaccurate estimate of the posterior distribution through the approximation-based approach such as FORM & SORM. Thus, it is very necessary to implement a very efficient reliability analysis method with the BUS algorithm to significantly reduce the number of evaluations to the likelihood function in Equation (12).

The surrogate model-based reliability analysis methods are known for their achievements in reducing the number of evaluations to the performance function as well as estimating accurate failure probability. However, it is hard in directly implementing surrogate model-based reliability analysis with limit state function in Equation (12) through SRS in BUS when the acceptance rate is extremely small. This point can be disclosed in the following section. To overcome this shortcoming, SRS with surrogate (SBU) is proposed in this paper.

3. Surrogate Model-Based Bayesian Updating

The concept of employing adaptive Kriging-based reliability analysis within the BUS framework involves the creation of a surrogate model, denoted as $\hat{h}(\mathbf{x}, p)$, to replace the computationally intensive limit state function $h(\mathbf{x}, p)$. This surrogate model allows for the efficient execution of SRS directly on it. In this section, we briefly review the essential elements of Kriging models and Kriging-based reliability analysis before delving into the challenges associated with integrating these methods with the BUS algorithm. As shown in Figure 1, the Kriging meta-model, also referred to as Gaussian Process Regression, is extensively utilized in the design of computer-based experiments.

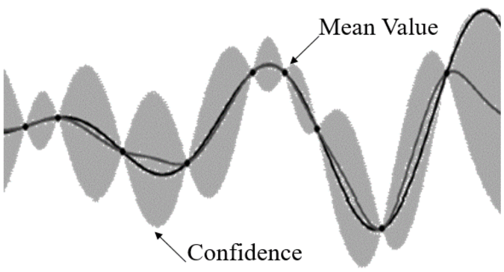

Figure 1. Conceptual illustration of Kriging surrogate model.

In this model, the predicted outcomes are characterized by both mean values and variances, adhering to a normal distribution. The Kriging model can be expressed as:

$$\hat{g}(\boldsymbol{X}) = F(\boldsymbol{\beta}, \boldsymbol{X}) + \psi(\boldsymbol{X}) = \boldsymbol{\beta}^T \boldsymbol{B}(\boldsymbol{X}) + gp(\boldsymbol{X}) \tag{14}$$

where $\boldsymbol{X}$ is the vector of random variables, $F(\boldsymbol{\beta}, \boldsymbol{X})$ represents the regression components, and $gp(\boldsymbol{x}_i)$ is the Gaussian process. The basis $\boldsymbol{B}(\boldsymbol{X})$ and the coefficients $\boldsymbol{\beta}$ take different forms depending on the complexity of the model, ranging from ordinary to quadratic. The Gaussian process $gp(\boldsymbol{X})$ is defined by its covariance matrix:

$$COV\left(gp(\boldsymbol{x}_i), gp(\boldsymbol{x}_j)\right) = \sigma^2 R\left(\boldsymbol{x}_i, \boldsymbol{x}_j; \boldsymbol{\theta}\right) \tag{15}$$

where σ^2 represents the process variance, and $R\left(\boldsymbol{x}_i, \boldsymbol{x}_j; \boldsymbol{\theta}\right)$ is the kernel function that models the correlation between observations, parameterized by $\boldsymbol{\theta}$. The Gaussian kernel function is specifically used here:

$$R\left(\boldsymbol{x}_i,\boldsymbol{x}_j;\boldsymbol{\theta}\right)=\prod_{k=1}^{N}\exp\left(-\theta^k\left(x_i^k-x_j^k\right)^2\right) \tag{16}$$

The parameters $\boldsymbol{\theta}$ are optimized via the Maximum Likelihood Estimation (MLE) method. Variations in $\boldsymbol{\theta}$ significantly impact the performance of the Kriging model, and this optimization is crucial to maintaining model accuracy:

$$\boldsymbol{\theta}=\underset{\theta^*}{\operatorname{argmin}}\left(\left|\boldsymbol{R}\left(\boldsymbol{x}_i,\boldsymbol{x}_j;\boldsymbol{\theta}^*\right)\right|^{\frac{1}{m}}\sigma^2\right) \tag{17}$$

The regression coefficients $\boldsymbol{\beta}$, and the estimated mean $\mu_{\hat{g}}(\boldsymbol{x})$ and variance $\sigma_{\hat{g}}^2(\boldsymbol{x})$ can be computed as follows:

$$\begin{gathered}\boldsymbol{\beta}=\left(\boldsymbol{F}^T\boldsymbol{R}^{-1}\boldsymbol{F}\right)^{-1}\boldsymbol{F}^T\boldsymbol{R}^{-1}\boldsymbol{Y}\\ \mu_{\hat{g}}(\boldsymbol{x})=\boldsymbol{B}^T(\boldsymbol{x})\boldsymbol{\beta}+\boldsymbol{r}^T(\boldsymbol{x})\boldsymbol{R}^{-1}\left(\boldsymbol{y}-\boldsymbol{F}\boldsymbol{\beta}\right)\\ \sigma_{\hat{g}}^2(\boldsymbol{x})=\sigma^2\left(\begin{array}{c}1-\boldsymbol{r}^T\left(\boldsymbol{x}_g\right)\boldsymbol{R}^{-1}\mathrm{r}\left(\boldsymbol{x}_g\right)+\\ \left(\boldsymbol{F}^T\boldsymbol{R}^{-1}\boldsymbol{r}(\boldsymbol{x})-\boldsymbol{B}(\boldsymbol{x})\right)^T\left(\boldsymbol{F}^T\boldsymbol{R}^{-1}\boldsymbol{F}\right)^{-1}\left(\boldsymbol{F}^T\boldsymbol{R}^{-1}\boldsymbol{r}(\boldsymbol{x})-\boldsymbol{B}(\boldsymbol{x})\right)\end{array}\right)\end{gathered} \tag{18}$$

Thus, the Kriging model output $\hat{g}(\boldsymbol{x})$ is not deterministic but probabilistic, represented by a normal distribution with mean $\mu_{\hat{g}}(\boldsymbol{x})$ and variance $\sigma_{\hat{g}}^2(\boldsymbol{x})$:

$$\hat{g}(\boldsymbol{x})\sim N\left(\mu_{\hat{g}}(\boldsymbol{x}),\,\sigma_{\hat{g}}^2(\boldsymbol{x})\right) \tag{19}$$

This stochastic nature facilitates the adaptive refinement of the Kriging model by strategically enriching the training dataset. In the ensuing sections, the proposed framework and its implementation steps are explained. Here, learning functions play a pivotal role in adaptively refining the Kriging model by iteratively selecting points based on the uncertainty associated with each design point. The popular U learning function, which evaluates the probability of making an incorrect sign estimation in $\hat{g}(\boldsymbol{x})$, is employed in this study. The U function is formulated as:

$$U(\boldsymbol{x})=\frac{\left|\mu_K(\boldsymbol{x})\right|}{\sigma_K(\boldsymbol{x})} \tag{20}$$

The general approach of adaptive Kriging-based reliability analysis begins with a small set of candidate design samples to estimate $\hat{\mathrm{P}}_{\mathrm{f}}$, subsequently refining the limit state as needed. The number of Monte Carlo Simulation (MCS) candidate design samples $\mathrm{N}_{\mathrm{MCS}}$ must be sufficient to ensure that the coefficient of variation $\mathrm{COV}_{\hat{\mathrm{P}}_{\mathrm{f}}}$ satisfies:

$$\mathrm{COV}_{\hat{\mathrm{P}}_{\mathrm{f}}}=\sqrt{\frac{1-\hat{\mathrm{P}}_{\mathrm{f}}}{\hat{\mathrm{P}}_{\mathrm{f}}\mathrm{N}_{\mathrm{MCS}}}}\leq\mathrm{COV}_{\mathrm{thr}} \tag{21}$$

Given this, the reliability of the final $\hat{\mathrm{P}}_{\mathrm{f}}$ and the limit state depends on ensuring $\mathrm{N}_{\mathrm{MCS}}$ is large enough so that $\mathrm{COV}_{\hat{\mathrm{P}}_{\mathrm{f}}}\leq\mathrm{COV}_{\mathrm{thr}}$. The integration of the Kriging surrogate model with the BUS algorithm thus enables effective Bayesian updating and model calibration.

4. Model Calibration of Motor FEM with Proposed Method

This section investigates the computational performance of the proposed model calibration method SBU and estimate its efficiency based on a FEM of induction motor. As shown in Figure 2, the finite element model is a four-pole, three-phase induction motor operating at a frequency of 50 Hz and a synchronous speed of 1500 rpm is a sophisticated computational technique employed to understand the motor's electromagnetic behavior under operational conditions. This motor type is a cornerstone in industrial applications due to its robustness, efficiency, and cost-effectiveness. The analysis utilizes ANSYS Maxwell 2D, a leading electromagnetic field simulation software, to model and simulate the motor's performance. Maxwell 2D is specifically designed for applications like rotating machines, transformers, and other electrical devices where precise electromagnetic field calculations are crucial. The choice of 2D analysis is particularly relevant here because it simplifies the computational process while still capturing the essential electromagnetic phenomena occurring in the motor.

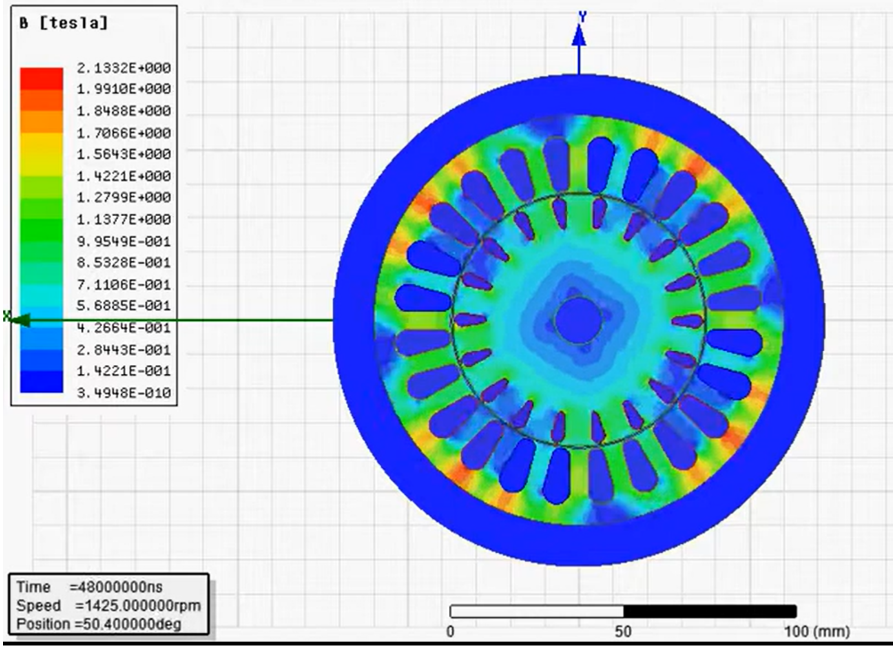

Figure 2. FEM of three-phase induction motor.

In the context of a four-pole, three-phase induction motor, FEA is employed to analyze several critical parameters, including magnetic flux distribution, eddy currents, core losses, and torque production. The motor's operation at 50 Hz frequency and a synchronous speed of 1500 rpm implies that it is likely designed for standard industrial power systems, making the analysis pertinent to real-world applications. The process begins with constructing a 2D model of the motor's geometry within the Maxwell environment. This model includes the stator, rotor, and air gap, accurately representing the motor's physical structure. Material properties, such as the permeability and conductivity of the core and windings, are defined based on real-world data or material specifications. Boundary conditions are then applied to simulate the motor's operational environment, including the supply voltage and frequency. Once the model is set up, the software solves Maxwell's equations, which govern the behavior of electric and magnetic fields in the motor. The simulation results provide a detailed picture of how the magnetic field interacts with the motor's components, allowing engineers to assess the distribution of magnetic flux within the stator and rotor cores. This is crucial for identifying areas of potential saturation, which can lead to inefficiencies or overheating.

By implementing Surrogate model-based Bayesian updating, the calibration process becomes significantly more efficient. This method reduces the number of necessary simulations from over 800 to just 30, an astonishing 96% reduction. The surrogate model, which acts as a computationally lightweight approximation of the FEM, allows for the exploration of the design space with minimal computational effort. It is constructed using a small set of FEM simulations and is then employed to predict the model's response under various parameter configurations. For instance, if each FEM simulation originally took 1 h to complete, the traditional approach would require approximately 800 h (or over 33 days) to finish the calibration process. However, with the surrogate model in place, the entire process is condensed to around 30 h, or just over a day—leading to a time savings of 32 days. This dramatic reduction in time also means that more design iterations can be evaluated within a given timeframe, enabling engineers to optimize motor designs more thoroughly.

In addition, the computational resources required for the FEM calibration are significantly lowered. If the original simulations demanded 10,000 CPU hours, the surrogate-based approach would only require 375 CPU hours. This not only reduces the load on high-performance computing resources but also allows engineers to allocate those resources to other critical projects. The accuracy of the motor FEM is maintained throughout the process, as the Bayesian updating refines the model parameters based on the predictions made by the surrogate model. This approach ensures that the surrogate model remains a reliable proxy for the full FEM, leading to a well-calibrated motor model that accurately reflects real-world performance. The success of this method in the motor FEM calibration case study underscores its potential applicability to other complex engineering problems, making it a powerful tool for efficient and accurate model calibration.

5. Conclusion

The case study on motor design, utilizing Finite Element Model Calibration combined with Surrogate Model-based Bayesian Updating, highlights a novel and efficient approach to enhancing the accuracy of electromagnetic simulations. This methodology addresses the inherent challenges of computational expense and uncertainty in traditional finite element analysis by integrating surrogate models, which approximate the behavior of the actual physical system with significantly reduced computational demands. The Bayesian Updating framework further refines these models by incorporating observed data, thereby continuously improving the predictive accuracy and reliability of the motor design. Through this approach, the study demonstrates that the calibration process becomes more adaptable and robust, allowing for real-time adjustments and optimizations. The practical implications are profound, particularly in the context of complex systems like electric motors, where precision in design directly correlates with performance, efficiency, and longevity. By reducing the computational burden and improving the fidelity of the simulations, the methodology enables more frequent and detailed analyses, ultimately leading to better-informed engineering decisions. In conclusion, the integration of surrogate models and Bayesian Updating in Finite Element Model Calibration not only streamlines the design process but also enhances the accuracy and reliability of simulation-based predictions. This advancement represents a significant step forward in the field of motor design and has broader applications in various engineering domains where simulation accuracy and computational efficiency are paramount. Future work can take a chance to explore the integration of Bayesian updating with machine learning techniques [49–58].

Funding

Not applicable.

Author Contributions

Conceptualization, G.Z. and T.Z.; writing—original draft preparation and writing—review and editing, G.Z. and T.Z. All of the authors read and agreed to the published the final manuscript.

Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

Data Availability Statement Statement

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

References

1 Xu H, Shi C, Fan W, *et al*. Improving Diversity and Discriminability Based Implicit Contrastive Learning for Unsupervised Domain Adaptation. *Applied Intelligence* 2024; **54**(**20**): 10007–100017.

2 Chen X, Wang M, Zhang H. Machine Learning-Based Fault Prediction and Diagnosis of Brushless Motors. *Engineering Advances* 2024; **4**(**3**): 130–142.

3 Beck JL, Au S-K. Bayesian Updating of Structural Models and Reliability Using Markov Chain Monte

Carlo Simulation. *Journal of Engineering Mechanics* 2002; **128**(**4**): 380–391.

4 Beck JL, Katafygiotis LS. Updating Models and Their Uncertainties. I: Bayesian Statistical Framework. *Journal of Engineering Mechanics* 1998; **124**(**4**): 455–461.

5 Giovanis DG, Papaioannou I, Straub D, *et al*. Bayesian Updating with Subset Simulation Using Artificial Neural Networks. *Computer Methods in Applied Mechanics and Engineering* 2017; **319**: 124–145.

6 Straub D, Papaioannou I. Bayesian Updating with Structural Reliability Methods. *Journal of Engineering Mechanics* 2015; **141**(**3**): 04014134.

7 Betz W, Papaioannou I, Beck JL, *et al*. Bayesian Inference with Subset Simulation: Strategies and Improvements. *Computer Methods in Applied Mechanics and Engineering* 2018; **331**: 72–93.

8 Ching J, Chen Y-C. Transitional Markov Chain Monte Carlo method for Bayesian Model Updating, Model Class Selection, and Model Averaging. *Journal of Engineering Mechanics* 2007; **133**(**7**): 816–832.

9 Betz W, Papaioannou I, Straub D. Transitional Markov Chain Monte Carlo: Observations and improvements. *Journal of Engineering Mechanics* 2016; **142**(**5**): 04016016.

10 Au S-K, Beck JL. Estimation of Small Failure Probabilities in High Dimensions by Subset Simulation. *Probabilistic Engineering Mechanics* 2001; **16**(**4**): 263–277.

11 Au S-K, Beck JL. Subset Simulation and Its Application to Seismic Risk Based on Dynamic Analysis. *Journal of Engineering Mechanics* 2003; **129**(**8**): 901–917.

12 Du S, Chen Z, Wu H, *et al*. Image Recommendation Algorithm Combined with Deep Neural Network Designed for Social Networks. *Complexity* 2021; **2021**(**1**): 5196190.

13 Gan Y, Zhu D. The Research on Intelligent News Advertisement Recommendation Algorithm Based on Prompt Learning in End-to-End Large Language Model Architecture. *Innovations in Applied Engineering and Technology* **2024**:1–19.

14 Straub D, Papaioannou I, Betz W. Bayesian Analysis of Rare Events. *Journal of Computational Physics* 2016; **314**: 538–56.

15 Schneider R, Thöns S, Straub D. Reliability Analysis and Updating of Deteriorating Systems with Subset Simulation. *Structural Safety* 2017; **64**: 20–36.

16 Rahrovani S, Au S-K, Abrahamsson T. Bayesian Treatment of Spatially-Varying Parameter Estimation Problems via Canonical BUS. In *Model Validation and Uncertainty Quantification*, *Volume 3: Proceedings of the 34th IMAC, A Conference and Exposition on Structural Dynamics 2016, Orlandoan, FL, USA, 25–28 January 2016*; Springer: Cham, Switzerlan, 2016.

17 Romero V, Swiler L, Giunta A. Construction of Response Surfaces Based on Progressive-Lattice-Sampling Experimental Designs with Application to Uncertainty Propagation. *Structural Safety* 2004; **26**(**2**): 201–219.

18 Giunta A, McFarland J, Swiler L, Eldred M. The promise and peril of uncertainty quantification using response surface approximations. *Structures and Infrastructure Engineering* 2006; **2**(**3–4**): 175–189.

19 Zhao W, Fan F, Wang W. Non-Linear Partial Least Squares Response Surface Method for Structural Reliability Analysis. *Reliability Engineering & System Safety* 2017; **161**: 69–77.

20 Wang Z, Zhao Y, Song C, *et al*. A New Interpretation on Structural Reliability Updating with Adaptive Batch Sampling-Based Subset Simulation. *Structural and Multidisciplinary Optimization* 2024; **67**(**1**): 7.

21 Zhu Y, Zhao Y, Song C, *et al*. Evolving Reliability Assessment of Systems Using Active Learning-Based Surrogate Modelling. *Physica D: Nonlinear Phenomena* 2024; **457**: 133957.

22 Blatman G, Sudret B. An Adaptive Algorithm to Build Up Sparse Polynomial Chaos Expansions for Stochastic Finite Element Analysis. *Probabilistic Engineering Mechanics* 2010; **25**(**2**): 183–197.

23 Dai H, Zhang H, Wang W, *et al*. Structural Reliability Assessment by Local Approximation of Limit State Functions Using Adaptive Markov Chain Simulation and Support Vector Regression. *Computer-Aided Civil and Infrastructure Engineering* 2012; **27**(**9**): 676–86.

24 Bourinet J-M. Rare-Event Probability Estimation with Adaptive Support Vector Regression Surrogates. *Reliability Engineering & System Safety* 2016; **150**: 210–221.

25 Echard B, Gayton N, Lemaire M. AK-MCS: An Active Learning Reliability Method Combining Kriging and Monte Carlo Simulation. *Structural Safety* 2011; **33**(**2**): 145–154.

26 Fauriat W, Gayton N. AK-SYS: An Adaptation of the AK-MCS Method for System Reliability. *Reliability Engineering & System Safety* 2014; **123**: 137–144.

27 Kaymaz I. Application of Kriging Method to Structural Reliability Problems. *Structural Safety* 2005; **27**(**2**): 133–151.

28 Gaspar B, Teixeira AP, Soares CG. Assessment of the Efficiency of Kriging Surrogate Models for Structural Reliability Analysis. *Probabilistic Engineering Mechanics* 2014; **37**: 24–34.

29 Wang Z, Shafieezadeh A. REAK: Reliability Analysis through Error Rate-Based Adaptive Kriging. *Reliability Engineering & System Safety* 2019; **182**: 33–45.

30 Wang Z, Shafieezadeh A. ESC: An Efficient Error-Based Stopping Criterion for Kriging-Based Reliability Analysis Methods. *Structural and Multidisciplinary Optimization* 2019; **59**: 1621–37.

31 Xiong S, Chen X, Zhang H. Deep Learning-BASED Multifunctional End-to-End Model for Optical Character Classification and Denoising. *Journal of Computational Methods in Engineering Applications* 2023; **3**: 1–13.

32 Xiong S, Zhang H, Wang M, Zhou N. Distributed Data Parallel Acceleration-Based Generative Adversarial Network for Fingerprint Generation. *Innovations in Applied Engineering and Technology* 2022; **1**: 1–12.

33 Hao Y, Chen Z, Jin J, Sun X. Joint Operation Planning of Drivers and Trucks for Semi-Autonomous Truck Platooning. *Transportmetrica A: Transport Science* 2023;1–37.

34 Hao Y, Chen Z, Sun X, Tong L. Planning of Truck Platooning for Road-Network Capacitated Vehicle Routing Problem. *arXiv* 2024; arXiv:240413512.

35 Tang Y. Investigating the Impact of Digital Transformation on Equity Financing: Empirical Evidence from Chinese A-Share Listed Enterprises. *Journal of Humanities, Arts and Social Science* 2024; **8**(**7**): 1620–1632.

36 Qiu Y, Wang J. A Machine Learning Approach to Credit Card Customer Segmentation for Economic Stability. In Proceedings of the 4th International Conference on Economic Management and Big Data Applications, ICEMBDA 2023, Tianjin, China; 27–29 October 2023.

37 Qiu Y. Financial Deepening and Economic Growth in Select Emerging Markets with Currency Board Systems: Theory and Evidence. *arXiv* 2024; arXiv:240600472.

38 Chen Z, Fu C, Wu R, *et al.* LGFat-RGCN: Faster Attention with Heterogeneous RGCN for Medical ICD Coding Generation. In Proceedings of the 31st ACM International Conference on Multimedia, Ottawa ON Canada, 29 October–3 November 2023; pp. 5428–5435. https://doi.org/10.1145/3581783.3612564.

39 Chen Z, Fu C, Tang X. Multi-domain Fake News Detection with Fuzzy Labels. In *Database Systems for Advanced Applications*. DASFAA 2023 International Workshops; Springer: Cham, Switzerland, 2023; Volume 13922, pp. 331–343. https://doi.org/10.1007/978-3-031-35415-1_23.

40 Yin, N, Wang, M, Chen, Z, *et al*. Dynamic spiking graph neural networks. In Proceedings of the AAAI Conference on Artificial Intelligence, Vancouver, BC, Canada, 20–27 February 2024; pp. 16495–16503. Available online: https://ojs.aaai.org/index.php/AAAI/article/view/29587 (accessed on 30 August 2024).

41 Su J, Chen Z, He C, *et al*. GSENet: Global Semantic Enhancement Network for Lane Detection. In Proceedings of the AAAI Conference on Artificial Intelligence, Vancouver, BC, Canada, 20–27 February 2024; pp. 15108–15116. Available online: https://ojs.aaai.org/index.php/AAAI/article/view/29433 (accessed on 30 August 2024).

42 Su H, Luo W, Liu D, *et al*. Sharpness-Aware Model-Agnostic Long-Tailed Domain Generalization. In Proceedings of the AAAI Conference on Artificial Intelligence, Vancouver, BC, Canada, 20–27 February 2024; pp. 15091–15099. Available online: https://ojs.aaai.org/index.php/AAAI/article/view/29431 (accessed on 30 August 2024).

43 Lu, L, Chen, Z, Lu, X, *et al*. Uniads: Universal Architecture-Distiller Search for Distillation Gap. In Proceedings of the AAAI Conference on Artificial Intelligence, Vancouver, BC, Canada, 20–27 February 2024; pp. 14167–14174. Available online: https://ojs.aaai.org/index.php/AAAI/article/view/29327 (accessed on 30 August 2024).

44 Chen J, Zou G, Zhou P, *et al*. Sparse Enhanced Network: An Adversarial Generation Method for Robust Augmentation in Sequential Recommendation. In Proceedings of the AAAI Conference on Artificial Intelligence, Vancouver, BC, Canada, 20–27 February 2024; pp. 8283–8291. Available online: https://ojs.aaai.org/index.php/AAAI/article/view/28669 (accessed on 30 August 2024).

45 Wang Y, Chen J, Wang M, *et al.* A Closer Look at Classifier in Adversarial Domain Generalization. In Proceedings of the 31st ACM International Conference on Multimedia, Ottawa ON Canada, 29 October–3 November 2023; pp. 280–289. https://doi.org/10.1145/3581783.3611743.

46 Wang M, Chen J, Wang Y, *et al*. Joint adversarial domain adaptation with structural graph alignment. *IEEE Transactions on Network Science and Engineering* 2023; **11**: 604–612.

47 Fu C, Chen Z, Shi J, *et al*. HAG: Hierarchical Attention with Graph Network for Dialogue Act Classification in Conversation. In Proceedings of the ICASSP 2023–2023 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Rhodes Island, Greece, 4–10 June 2023; pp. 1–5. Available online: https://ieeexplore.ieee.org/abstract/document/10096805/ (accessed on 30 August 2024).

48 Gu Y, Yan D, Yan, S, *et al*. Price Forecast with High-Frequency Finance Data: An Autoregressive Recurrent Neural Network Model with Technical Indicators. In Proceedings of the 29th ACM International Conference on Information & Knowledge Management, Virtual Event, Ireland, 19–23 October 2020; pp 2485–2492. https://doi.org/10.1145/3340531.3412738.

49 Gu Y, Chen K. GAN-Based Domain Inference Attack. In Proceedings of the AAAI Conference on Artificial Intelligence, Washington, DC, USA, 7–14 February 2023; Volume 37. https://doi.org/10.1609/aaai.v37i12.26663.

50 Gu Y, Sharma S, Chen K. Image Disguising for Scalable GPU-accelerated Confidential Deep Learning. In Proceedings of the 2023 ACM SIGSAC Conference on Computer and Communications Security, Copenhagen, Denmark, 26–30 November 2023; pp 3679–3681. Available online: https://dl.acm.org/doi/abs/10.1145/3576915.3624364 (accessed on 29 May 2024).

51 Xu H, Shi C, Fan W, *et al.* Improving diversity and discriminability based implicit contrastive learning for unsupervised domain adaptation. *Applied Intelligence* 2024; **54**: 10007–10017. https://doi. org/10.1007/s10489-024-05351-y.

52 Jiang Z, Zhang L, Liang X, *et al*. CbDA: Contrastive-Based Data Augmentation for Domain Generalization. *IEEE Transactions on Computational Social Systems* 2024; 1–8. Available online: https://ieeexplore. ieee.org/abstract/document/10601637/(accessed on30August2024).

53 Li, B, Ma, Y, Liu, Y, *et al*. Federated Learning on Distributed Graphs Considering Multiple Heterogeneities. In Proceedings of the ICASSP 2024–2024 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Seoul, Korea, 14–19 April 2024; pp 5140–5144. Available online: https://ieeexplore.ieee.org/abstract/document/10447691/ (accessed on 30 August 2024).

54 Xie T, Luo S, Chen Z, *et al*. Semanticmapper: Region-Specific Domain Adaptation for 3D Shapes Through Lexical Delineation. In Proceedings of the ICASSP 2024–2024 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Seoul, Korea, 14–19 April 2024; pp 3950–3954. Available online: https://ieeexplore.ieee.org/abstract/document/10446758/ (accessed on 30 August 2024).

55 Luo S, Jiang Z, Chen Z; *et al.* Domain Adaptive Graph Classification. In Proceedings of the ICASSP 2024–2024 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Seoul, Korea, 14–19 April 2024; pp 4915–4919. Available online: https://ieeexplore.ieee.org/abstract/document/10448226/ (accessed on 30 August 2024).

56 Tang X, Tian H, Chen Z, *et al.* Learning to Represent Patches. In Proceedings of the 2024 IEEE/ACM 46th International Conference on Software Engineering: Companion Proceedings, Lisbon, Portugal, 14–20 April 2024; pp. 396–397. https://doi.org/10.1145/3639478.3643521.

57 Du S, Chen Z, Wu H, *et al*. Image Recommendation Algorithm Combined with Deep Neural Network Designed for Social Networks. *Complexity* 2021; **2021**: 5196190. https://doi.org/10.1155/2021/5196190.

58 Wang Y, Chen Z, Fu C. Synergy Masks of Domain Attribute Model DaBERT: Emotional Tracking on Time-varying Virtual Space *C*ommunication. *Sensors* 2022; **22**: 8450.