

# Measurement, Divergence, and Cultivation Paths of Mathematical Literacy Among Higher Vocational Students Empowered by Generative AI

Xuezhen Liang <sup>1</sup>, Dongping Wei <sup>1</sup>, Xinyue Zhang <sup>2</sup>, Le Zhang <sup>2,\*</sup> and Huali Chen <sup>1,\*</sup>

<sup>1</sup> *Institute of Applied Mathematics, Shenzhen Polytechnic University, Shenzhen 518055, China*

<sup>2</sup> *School of Chemical Engineering, Guangdong University of Petrochemical Technology, Maoming 525000, China*

**Abstract:** With the explosive development of Generative Artificial Intelligence (GenAI), the mathematical teaching paradigm in higher vocational education is undergoing an unprecedented restructuring. As the cornerstone of core competencies, mathematical literacy exhibits new characterizing dimensions in an AI-empowered environment. Based on the characteristics of mathematics as a discipline, this study constructs the “ABCE” analytical framework encompassing Affective Experience, Behavioral Engagement, Cognitive Criticism, and Ethical Awareness. Through an empirical investigation of 1,086 students from a high-level applied university in China, utilizing correlation, ANOVA, and cluster analysis, the findings indicate that students demonstrate an imbalanced profile of “high affective identification, moderate ethical awareness, and low behavioral efficacy.” Mathematical foundation and technology acceptance are the core drivers of literacy divergence, and a significant contradiction exists between “cognitive inertia” and “tool dependency.” Based on these findings, four typical learner profiles are identified, and a stratified pedagogical intervention strategy focusing on the deep integration of “Human-AI-Teacher” is proposed to provide theoretical support and empirical evidence for the reform of mathematical evaluation in vocational education.

**Keywords:** Generative Artificial Intelligence (GenAI); higher vocational education; mathematical literacy; human-AI collaboration; ABCE framework

## 1. Introduction

**Research Background and Problem Statement** In the wave of digital transformation, Generative AI (GenAI), represented by ChatGPT and Claude, has not only changed information retrieval but also profoundly intervened in human logical reasoning and problem-solving processes. As hubs for cultivating technical and skilled talents, higher vocational institutions are experiencing a paradigm shift in mathematics from “knowledge transmission” to “competency-oriented evolution”. Mathematical literacy is no longer limited to calculation and logic; it now encompasses the comprehensive ability to construct models, verify results, and exercise ethical criticism with the assistance of intelligent tools.

However, the definition of “AI Mathematical Literacy” remains vague in academia. Especially in vocational education, there is a lack of systematic empirical research on cognitive bias, affective anxiety, and behavioral maladjustment when students use AI for mathematical learning. This study aims to answer: What is the structure

of AI mathematical literacy among vocational students? What are the characteristics of literacy divergence among different groups? How can a scientific cultivation path be constructed to optimize human-AI synergy?

Since 2020, research on AI literacy has evolved from foundational competency definitions [1] into a multidimensional framework encompassing affective, behavioral, cognitive, and ethical dimensions [2,3]. However, existing scholarship predominantly focuses on generic contexts [4], leaving a research gap in “discipline-specific” demands within fields such as higher vocational mathematics. To bridge this gap, this study integrates the PISA mathematical literacy framework, the Technology Acceptance Model [5], and Constructivism, supplemented by Self-Determination Theory [6], Achievement Emotion Theory [7], Bloom’s Revised Taxonomy and metacognitive framework [8,9]. We propose a four-dimensional analytical model: “Affective Experience, Behavioral Engagement, Cognitive Critique, and Ethical Awareness”. This framework investigates how vocational students leverage AI to enhance mathematical modeling and logical deduction—improving technical fluency while fostering high-order cognitive construction through “evidenced skepticism” and establishing ethical boundaries for “human-centric, machine-assisted” symbiosis. By doing so, this study shifts the paradigm from generic AI literacy to discipline-specific competence, providing theoretically robust support for mathematical pedagogical reform in the vocational education sector during the intelligence era.

## 2. Theoretical Framework and Dimension Construction

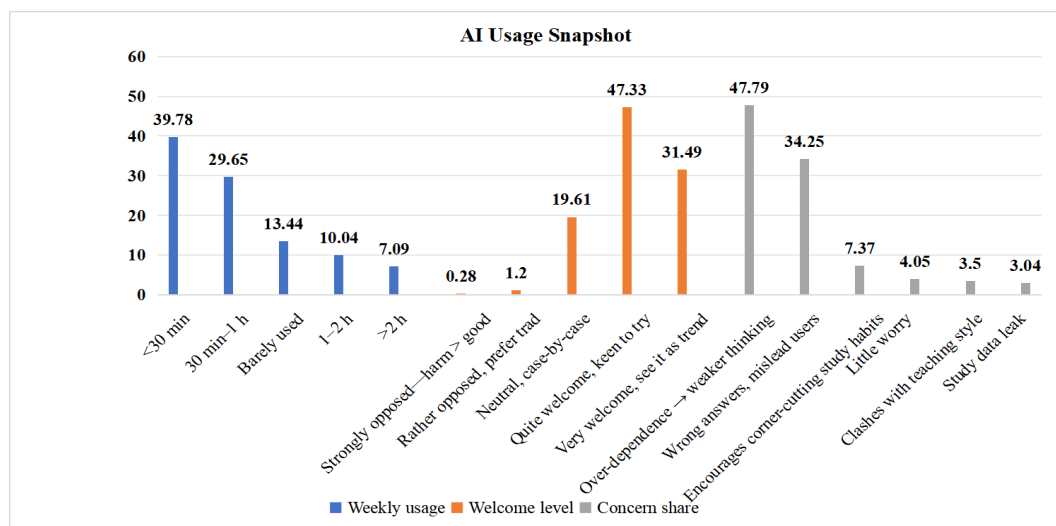
Integrating Self-Determination Theory (SDT) and the Technology Acceptance Model (TAM), this study proposes a four-dimensional “ABCE” model (as shown in Figure 1):

**(1) Affective Experience:** Refers to students’ emotional states during AI interaction, including academic self-efficacy, tool anxiety, and autonomous motivation.

**(2) Behavioral Engagement:** Examines the frequency of interaction, the application of Prompt Engineering, and the depth of using AI for knowledge tracing.

**(3) Cognitive Criticism:** The core of AI mathematical literacy, referring to the ability to verify, correct, and optimize AI-generated mathematical logic and reasoning.

**(4) Ethical Awareness:** Involves adherence to academic integrity, vigilance against algorithmic bias, and the scientific definition of human-machine boundaries.



**Figure 1.** Framework of the ABCE Model for AI Mathematical Literacy.

## 3. Methodology

**(A) Participants.** The sample was drawn from a representative high-level vocational university. Using stratified random sampling, the survey covered majors in Intelligent Manufacturing, Electronic Information, and Economics/Management. A total of 1,086 valid questionnaires were recovered, with an effective rate of 98.7% (as shown in Table 1).

(B) Instrumentation. The scale was developed based on the AI Literacy Scale by Ng et al., adapted for mathematics. It consists of 20 items using a 5-point Likert scale.

(C) Reliability and Validity. The Cronbach's Alpha for the total scale was 0.978, and sub-dimension coefficients were all above 0.85. The KMO value was 0.976, and the Bartlett test was significant ( $p < 0.001$ ), indicating excellent statistical validity (as shown in Table 2).

**Table 1.** Demographic Characteristics of the Sample.

Variable	Category	Percentage (%)
Gender	Male	58.64%
	Female	41.36%
Major Field	Intelligent Manufacturing	40.82%
	Electronic Information	23.18%
	Economics & Management	17.73%
	Civil Engineering	4.27%
	Others	14.00%
	first-year undergraduate students	98.7%
	second-year undergraduate students	0.82%
Math Foundation	third-year undergraduate students	0.45%
	Excellent	10.27%
	Good	39.82%
	Moderate	31.82%
	Weak	12.09%
	Poor	6.00%

**Table 2.** Reliability and Factor Analysis Results.

Dimension	No. of Items	Cronbach's Alpha
Affective (A)	5	0.935
Behavioral (B)	4	0.917
Cognitive (C)	6	0.951
Ethical (E)	5	0.941
Total Scale	20	0.978

#### 4. Results and Findings

(A) The survey revealed that students scored highest in the ethical awareness dimension (78.0%), demonstrating strong academic integrity. However, their performance lagged in behavioral engagement (75.0%) and emotional experience (75.5%). This indicates that while students are willing and eager to use AI, their confidence in learning and proficiency in utilizing AI still require improvement.

(B) Deep Insight into Usage Behavior:

**(1) Tool Preference:** Among the problems students most hope AI can solve, respondents prioritized the following in order: guiding problem-solving approaches (81.49%), tracing knowledge points (68.14%), applying knowledge to similar problems (61.6%), instant problem-solving (50.46%), video/animation explanations (41.62%), and personalized error notebooks (41.44%). The demand for problem-solving guidance was the highest, far exceeding that for instant solutions. This indicates students prefer AI tools with strong logical reasoning capabilities to inspire their own problem-solving approaches rather than merely obtaining answers.

**(2) Duration and Effects:** There is an inverted U-shaped relationship between usage duration and cognitive

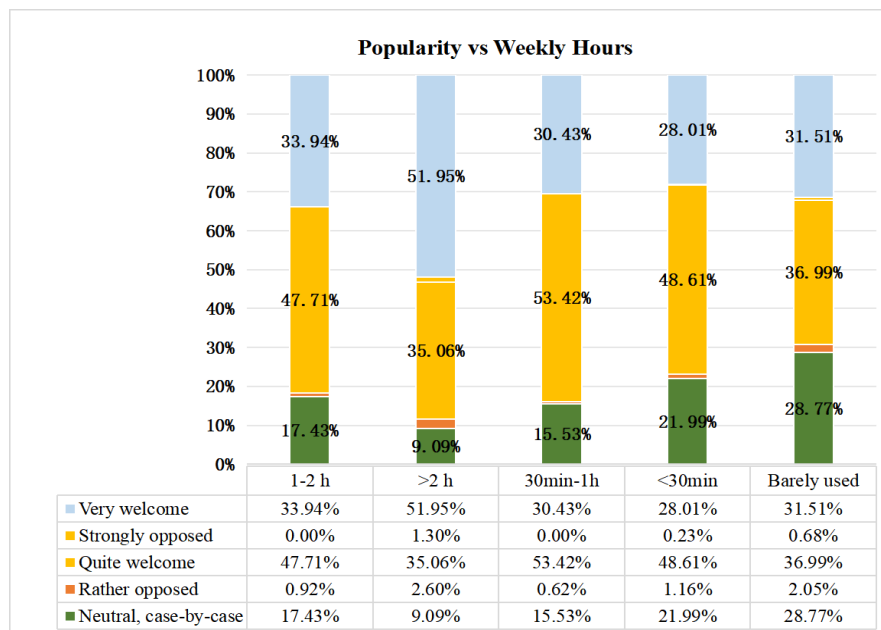
load. Students who use it for 60 min or less per week (nearly 70%, as shown in Figure 2) demonstrate the highest sense of efficacy.

### (C) Divergence Testing

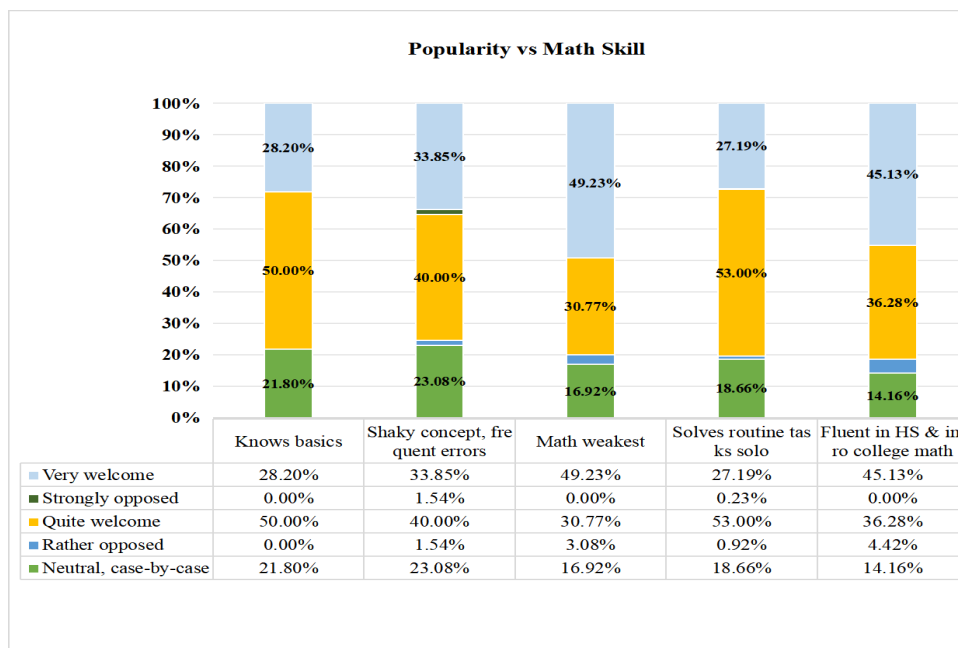
**(1) Impact of Math Foundation:** ANOVA proved that students with strong foundations performed significantly better in Cognitive Criticism ( $F = 45.2$ ,  $p < 0.001$ ). This reflects the “Matthew Effect” where technology does not automatically bridge the cognitive gap (as shown in Figure 3).

**(2) Major Differences:** STEM students showed higher exploratory willingness, while Liberal Arts and Management students excelled in ethical prudence.

Furthermore, the factor loadings (as shown in Table 3) further validated the scientific rigor of the questionnaire design, providing support for the reliability of the relevant data. Additionally, a single-factor analysis of variance (ANOVA) for group variables was employed to examine and confirm the significant influence of the variables on the four dimensions of AI mathematical literacy (as shown in Table 4).



**Figure 2.** Comparison of the Impact of Key Variables on AI Acceptance (I).



**Figure 3.** Comparison of the Impact of Key Variables on AI Acceptance (II).

**Table 3.** Items and Factor Loadings of the AI Mathematics Literacy Scale.

Dimension (Factor)	Optimized Items	Factor Loading
Factor 1: Affective Experience	1. AI tools have enhanced my interest in learning mathematics.	0.74
	2. The interactivity of AI makes the process of learning mathematics more engaging.	0.74
	3. Prompt feedback from AI has boosted my confidence in learning mathematics.	0.74
	4. Using AI effectively alleviates the pressure and anxiety associated with learning math.	0.74
	5. AI helps me better understand the practical application value of mathematics.	0.66
Factor 4: Behavioral Strategy	1. Using AI to obtain multiple solutions or similar problems for intensive practice.	0.76
	2. Cross-checking the accuracy of AI answers against textbooks and personal notes.	0.67
	3. Utilizing AI visualization features to deepen understanding of abstract concepts.	0.51
	4. Using AI to assist in building a systematic mathematical knowledge network.	0.61
Factor 3: Cognitive Criticism	1. AI has stimulated my divergent thinking and creativity in mathematics.	0.50
	2. AI assistance has improved the efficiency of solving complex mathematical problems.	0.58
	3. Remaining cautious about the solution steps provided by AI and not accepting them without question.	0.72
	4. Critically evaluating the rationality and logical rigor of AI-generated solutions.	0.69
	5. Being able to decompose mathematical tasks and define the division of labor between humans and AI.	0.63
Factor 2: Ethical Awareness	1. Defining the boundaries of AI use and consciously maintaining academic integrity.	0.60
	2. Proactively declaring when AI is used to assist in completing assignments or projects.	0.70
	3. Agreeing that schools have a responsibility to ensure students have equitable access to AI resources.	0.78
	4. Agreeing that future mathematics education should involve deep human-AI integration.	0.78
	5. Reflecting on how to maintain core thinking skills while utilizing AI tools.	0.65

**Table 4.** One-Way ANOVA for Group Variables Related to AI Mathematical Literacy.

Grouping Variables	Stat.	Affective	Behavioral	Cognitive	Ethical
Gender	F	13.357	17.371	20.680	13.485
	Sig.	0.000 ***	0.000 ***	0.000 ***	0.000 ***
Math Foundation	F	7.890	9.924	12.459	9.440
	Sig.	0.000 ***	0.000 ***	0.000 ***	0.000 ***
AI Usage Time	F	5.517	3.264	4.512	5.116
	Sig.	0.000 ***	0.011 *	0.001 **	0.000 ***
Attitude toward AI	F	31.348	26.469	27.384	29.063
	Sig.	0.000 ***	0.000 ***	0.000 ***	0.000 ***
Perceived Concerns	F	3.538	1.010	1.845	4.512
	Sig.	0.004 **	0.411	0.102	0.378

Note: \*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ .

## 5. Learner Profiles: Cluster Analysis

Using the K-means algorithm, students were classified into four groups (as shown in Table 5):

**(1) High-Engagement Beneficiaries (20.6%):** High engagement and high criticism; these students master AI as a cognitive partner.

- (2) **Mid-High Growth (43.1%)**: Positive attitude, currently in a capability-climbing phase.
- (3) **Mid-Conflict (34.8%)**: High identification but low behavioral capability; prone to cognitive inertia.
- (4) **Low-Perception Disengaged (1.5%)**: Indifferent or resistant to the technological shift.

**Table 5.** Characteristics and Intervention Strategies for Four Typical Groups.

Learner Profile	%	Affective Experience (A)	Behavioral Engagement (B)	Cognitive Criticism (C)	Ethical Awareness (E)	Suggested Strategy
High-Engagement Beneficiaries	20.6 %	2.38	2.25	2.22	3.16	Cultivate as “AI Peer Mentors”
Mid-High Growth	43.1 %	4.89	4.91	4.91	4.92	Strengthen logical verification training.
Mid-Conflict	34.8 %	3.08	3.07	3.13	3.18	Use heuristic teaching to break bottlenecks.
Low-Perception Disengaged	1.5%	3.85	3.86	3.93	4.02	Basic literacy popularization.

## 6. Discussion and Recommendations

(A) Theoretical Implications: From “Technology Substitution” to “Cognitive Symbiosis” The improvement of AI literacy is a process of cognitive restructuring. AI should be viewed not just as a tool but as a “scaffold” for thinking.

(B) Practical Strategies: Stratified Cultivation:

(1) **Curriculum Restructuring**: Integrate “Prompt Engineering” with mathematical logic. Introduce “Error Correction Tasks” in assessments to enhance cognitive criticism.

(2) **Ethical Guidance**: Set clear boundaries for human-AI collaboration, shifting focus from “finding answers” to “seeking explanations”.

(3) **Platform Optimization**: Develop “Education-friendly” AI assistants with step-by-step heuristic functions to prevent cognitive atrophy.

## 7. Conclusions

This study outlines the landscape of AI mathematical literacy in vocational education. Future education should focus on high-order thinking in intelligent environments, helping every student achieve a cognitive leap in the era of human-AI collaboration.

## Funding

This research was funded by Guangdong Province Undergraduate Teaching Quality and Teaching Reform Construction Project (Higher Education Teaching Reform Project: 2024-30-884). Quality Engineering Project of Shenzhen Polytechnic University (General Project: 1005-0452). Smart Course Project of Guangdong University of Petrochemical Technology: 2024-59. Projects of Talents Recruitment of GDUPT:2020rc039.

## Author Contributions

Writing—original draft, X.L., D.W., X.Z., L.Z. and H.C.; writing—review and editing, X.L., D.W., X.Z., L.Z. and H.C. All authors have read and agreed to the published version of the manuscript.

## Institutional Review Board Statement

Not applicable.

### Informed Consent Statement

Not applicable.

### Data Availability Statement

Not applicable.

### Conflicts of Interest

The authors declare no conflict of interest.

### References

- 1 Long D, Magerko B. What Is AI literacy? Competencies and Design Consideration. In Proceedings of the 2020 CHI Conference on Human Factors in Computing Systems, Honolulu, HI, USA, 25–30 April 2020; pp. 1–16.
- 2 Ng DTK, Leung JKL, Chu KWS, *et al.* AI Literacy: Definition, Teaching, Evaluation and Ethical Issues. *Proceedings of the Association for Information Science and Technology* 2021; **58(1)**: 504–509.
- 3 Ng DTK, Wu W, Leung JKL, *et al.* Design and Validation of the AI Literacy Questionnaire: The Affective, Behavioural, Cognitive and Ethical Approach. *British Journal of Educational Technology* 2024; **55(3)**: 1082–1104.
- 4 Zhou X, Li Y, Chai CS, *et al.* Defining, Enhancing, and Assessing Artificial Intelligence Literacy and Competency in K-12 Education from a Systematic Review. *Interactive Learning Environments* 2025; 1–23.
- 5 Davis FD. Perceived Usefulness, Perceived Ease of Use, and User Acceptance of Information Technology. *MIS Quarterly* 1989; **13**: 319–340.
- 6 Deci EL, Ryan RM. *Intrinsic Motivation and Self-Determination in Human Behavior*; Springer Science & Business Media: New York, NY, USA, 2013.
- 7 Pekrun R. The Control-Value Theory of Achievement Emotions: Assumptions, Corollaries, and Implications for Educational Research and Practice. *Educational Psychology Review* 2006; **18(4)**: 315–341.
- 8 Bloom BS. *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*; Longman: New York, NY, USA, 2001.
- 9 Flavell JH. Metacognition and Cognitive Monitoring: A New Area of Cognitive–Developmental Inquiry. *American Psychologist* 1979; **34(10)**: 906.

