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Article

Research on the Testing Duration of Planning Model

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Abstract: Reliability growth plans are typically described by using reliability growth planning models that generate growth planning curves. The PM2 model, known as the planning model based on projection methodology, is a widely used growth planning model. The PM2 model can always meet the assigned reliability target during any reliability growth testing duration, regardless of changes in the management strategy parameter and/or the fixed effectiveness factor parameter. In fact, these two parameters determine whether reliability growth testing is successful or not and how long it takes. That is, the PM2 model currently in place needs to be modified accordingly. Firstly, a sensitivity analysis is carried out on the three parameters of the PM2 model, followed by a discussion on the correlation between the product of two process management parameters and the duration of testing. Then, a relationship equation for the testing duration is constructed using these two process management parameters. Finally, an extended model for planning reliability growth has been proposed. In addition, the proposed planning model is utilized to examine an illustrated example and a real case. The results have demonstrated that the model is reasonable and reliable, as the testing duration, was removed from the reliability growth planning model.

Keywords: reliability growth; PM2 model; nonlinear equation; engineering management; testing duration; sensitivity analysis

1. Introduction

It is obvious that a complex system or product requires high reliability. This is usually achieved through a process of increasing reliability. Reliability growth programs are important because making design changes after release is costly. Reliability growth planning is a significant segment of the reliability growth program. During the development stage, the goal of planning is to create a dependable growth curve that can be reached. The reliability tests judge the progress of reliability growth in single or multiple test stages by using the temporary milestones provided by the reliability growth curves. At the end of every test stage, the reliability of a complicated system is measured and compared with the corresponding values. If the assessed reliability indexes are higher than or exceed the milestone values, the reliability growth testing continues. If the assessed reliability indexes are lower than the milestone values, the reliability growth programs need to be improved [1]. Reliability growth plans, which involve growth planning curves, are often described using reliability growth planning models [2].

Significant research on reliability growth planning models has already done for decades. Weiss tried to determine the level of reliability, whether the reliability is increasing, and if so, the rate of increase, and the

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expected final reliability of complex systems, where the failures and operating time followed a Poisson distribution. Then methods were developed to match experimental data with reliability growth curves. Additionally, certain mathematical models were suggested to choose a suitable growth curve for an idealized engineering development process [3]. To find the relationship between the cumulative operating hours with the cumulative failure rates, Duane conducted a study showing that they fell very close to a straight line when plotted on the double-log paper. These measurements were based on the fault data of five different products during their development stages. It was concluded that multiple complex systems are being developed with similar improvement rates. To control development processes and predict growth patterns, a learning curve method was suggested [4]. To assess the future MTBFs (mean time between failures) of systems on the supposed growth rates, a planning tool including Duane postulate was built by Selby and Miller [5], and the U. S. Department of Defense [6].

Ellner and Wald put forward the AMSAA (army materiel system analysis activity) maturity projection model (AMPM model) [7], where only the first surfaced times of occurrences observed in the development test are utilized, so the model can adapt to any corrective action strategies. Through shrinkage estimation technology, Ellner and Hall improved the AMPM-stein model by postponing the corrective actions when the test was ended, thus higher estimation accuracy were obtained [8]. To accommodate arbitrary corrective strategies, Crow developed an extended model that combines the Crow tracking model and the Crow projection mode [9]. To estimate the discrete system reliability based on planning methodology, a full Bayesian estimation was proposed by Strunz and Herrmann by considering all components, subsystems, and system-level test data [10]. Based on essential function failures (EFF), a novel method for planning system reliability growth was proposed by Bell and Bearden by taking into account the management strategy [11]. In terms of the posterior distribution of the system failure function, a Bayesian projection model was proposed by Wayne and Modarres on the basic of the AMPM model [12]. For widely adapting to various management strategies, the multi-phase reliability growth model (MPRGM) was provided by Jackson [13]. To intelligently allocate expensive and limited resources in the early product-development stage, a multi-objective and multi-stage reliability growth planning method was proposed by Li, Mobin, and Keyser [14]. By investigating the interactions between testing time, corrective actions, and latent failures, a life-cycle reliability growth model was presented by Jin and Li [15]. To maximize system reliability in the test planning, Heydari and Sullivan contributed a new model for a series-parallel system under limited resources by merging testing components method and installing redundancies [16]. Based on the Crow projection model, an expanded growth planning model for discrete systems wan provided [17]. Detailed information on the kinds of reliability growth planning models can be found in MIL-HDBK-189 conducted by Department of Defense, which includes assessing plans, testing requirements, technical resource requirements, and so on.

Crow found that planning models are influenced by their inherent parameters, as different parameters create unique growth curves [18]. Some significant research has also been studied on the model parameters. Ellner and Trapnell studied the fixed effectiveness factors of army systems under the reliability growth test and concluded that the overall average is about 0.69 [19]. Assuming that the reliability growth rates for all components conform to the AMSAA model, the Coit model was presented which allocated the reliability growth testing time of components in series as a whole [20]. Jin and Wang provided three validity functions to express corrective actions and applied them to failure intensity models for projecting the mean time between failures (MTBF) of systems, a two-objective reliability growth model was provided for maximizing the reliability of the system and minimizing the uncertainty of reliability under limited corrective actions [21]. Calomeris and Herbert [22] pointed out that the experiment duration, primary MTBF, and goal MTBF all have a big influence on the growth pattern, but no further research has been found. Bell and Bearden emphasized that the management strategy parameter is critical for reliability growth planning [11]. Crow recommended the management strategy should be 0.95 for the new and unproven systems [18]. Awad stated that the test duration will be affected by the initial failure rate. Moreover, the sensitivity analyses of the hypothetical parameters were demonstrated. Finally, a new method to estimate the initial reliability of the system was proposed [23]. Considering the effectiveness of proposed corrective actions, Honecker and Yenal studied the scale factor methods and fractional failure methods, then a mathematical relationship expressed by the scale factor and the corrective action effectiveness was proposed [24]. Seo discovered that when developing reliability growth plans for a guided weapon system using the PM2-Continuous Model [25], the reliability growth requirements are influenced by several factors. These factors include the initial MTBF to target MTBF ratio, the management strategy ratio, the average fixed effectiveness factor, and the development testing duration. To predict the reliability of weapon systems using the PM2-Extended Model, a simulation method is proposed for estimating the average FEF (fixed effectiveness factor) [26]. Based on some specific experiment data, Li et al. studied how the PM2 planning curves are influenced by the model parameters [27].

Brown stated that two methods are utilized to structure a reliability growth planning curve (RGPC) [28]. The first method involves expert information from similar projects to represent an expectation of growth. The second method is a planned RGPC on the basis of program milestones, in which, the management department determines the growth rate by establishing the growth targets that the test must meet the acceptable testing schedule. Once each test phase is completed, the realized growth curve is compared with the program milestones to determine whether to reallocate resources. The planning model based on projection methodology (PM2) is a popular reliability growth planning model in terms of the above second method. The PM2 model has been widely used by the Department of Defense for reliability growth planning. PM2 model has become a particular requirement for the U.S. army systems since June 2009 [11]. The essential characteristic of the PM2 model is that the planning curves always meet the assigned reliability targets for any assigned testing duration.

However, the reliability growth planning model needs to describe process management parameters change, such as the fixed effectiveness factor and the management strategy [28]. To improve the universality of the PM2 model, a discrimination inequality was constructed for the product of the management strategy and the average fixed effectiveness factor, then the specific lower limit of the product was studied [29].

In summary, the PM2 model that is currently in use has a number of uncertain parameters. These parameters limit planning rationality due to various human factors. The testing duration will be affected by the fixed effectiveness factor and the management strategy in terms of statistical data. However, the current literature does not provide any research on testing duration, and it is only used as a subjective input so far. Therefore, the negative correlation between the planned testing duration with the fixed effectiveness factor and management strategy will be discussed, depending on the specific lower limit of the testing duration. After that, a nonlinear equation for testing duration that incorporates the fixed effectiveness factor and management strategy is presented. Finally, a model for growth planning that uses the nonlinear equation and eliminates the subjective input of testing duration is proposed.

2. PM2 Model and Parameters

To obtain system-level reliability growth planning curves, the PM2 model for continuous systems is provided by incorporating the corrective action strategy and developmental test schedule. The related terminologies are provided below.

(i) d, the average fixed effectiveness factor of failure modes, is the reduced fraction of an initial failure mode due to the implementation of corrective actions. Normally, the planning curves will change accordingly once the fixed effectiveness coefficient changes. According to experimental and historical data, the common range of the parameter is [0.6, 0.8].

(ii) MS, the management strategy, which is the fraction of the primary failure intensity because of failure modes that would be corrected if occurred during the developmental test (also named ‘B modes’, the modes if surfaced will not be corrected are named ‘A modes’), is expressed by:

$$MS = \frac{\lambda_B}{\lambda_A + \lambda_B} \tag{1}$$

where λ_A is the failure intensity of A modes, λ_B is the failure intensity of B modes. Crow concludes that the system reliability will grow faster when the MS is larger [30]. That is, the reliability growth trend is directly influenced by MS. Equally, the common range of MS is assigned as [0.7, 0.95].

(iii) M_I, the initial MTBF of the system, which is defined as:

$$M_I = \frac{1}{\lambda_I} = \frac{1}{\lambda_A + \lambda_B} \tag{2}$$

where λ_I is the initial system failure intensity, consisting of A modes and B modes.

(iv) M_F, the final MTBF achieved at the end of the reliability growth test.

(v) M_{GP}, the growth potential of system MTBF, is defined as

$$M_{GP} = \frac{M_I}{1 - MS \cdot d} \tag{3}$$

(vi) *GPDM*, the growth potential design margin, which can be designed as

$$GPDM = \frac{M_{GP}}{M_F} \tag{4}$$

According to historical experience and experimental data, the maximum of *GPDM* is expected as 1.5. According to Equations (3) and (4), the boundary of the testing growth rate (M_F/M_I) can be obtained as [1.15, 2.78]. If the required testing growth rate is beyond this range, the reliability growth test should be carried out in batches.

(vii) $\lambda(t)$, the projected system failure function at test time t, is expressed as:

$$\lambda(t) = \lambda_A + (1-d)\lambda_B + d \cdot h(t) \tag{5}$$

where $h(t)$ is the incidence function of new observed B-modes at time t, which can be expressed as

$$h_{PM2}(t) = \frac{\lambda_B}{1 + \beta_{PM2} \cdot t} \tag{6}$$

where β_{PM2} is the scale parameter of $h_{PM2}(t)$, is given by:

$$\beta_{PM2} = \frac{1}{T} \cdot \frac{1 - \dfrac{M_I}{M_F}}{d \cdot MS - \left(1 - \dfrac{M_I}{M_F}\right)} \tag{7}$$

where parameter T is the developmental testing duration, which will be inputted directly by management department. Thus

$$\lambda(t) = \lambda_A + (1-d)\lambda_B + d \cdot \frac{\lambda_B}{1 + \left(\dfrac{1}{T} \cdot \dfrac{1 - \dfrac{M_I}{M_F}}{d \cdot MS - \left(1 - \dfrac{M_I}{M_F}\right)}\right) \cdot t} \tag{8}$$

In terms of Equations (1) and (2), λ_A and λ_B can be expressed as:

$$\begin{cases} \lambda_A = (1 - MS) \cdot \lambda_I \\ \lambda_B = MS \cdot \lambda_I \end{cases} \tag{9}$$

For a given target value, the transformed equation of Equation (8) is expressed as:

$$\frac{1}{M_F} = \frac{1 - dMS}{M_I} + \frac{\dfrac{dMS}{M_I}}{1 + \dfrac{t}{T} \cdot \dfrac{M_F - M_I}{dMSM_F - (M_F - M_I)}}$$

Further, the below expression can be obtained

$$\begin{aligned} &\frac{1}{M_F} - \frac{1 - dMS}{M_I} = \frac{\dfrac{dMS}{M_I}}{1 + \dfrac{t}{T} \cdot \dfrac{M_F - M_I}{dMSM_F - (M_F - M_I)}} \\ &\Rightarrow M_I(M_F - M_I)\left(M_I - M_F + M_F MSd\right)t = M_I(M_F - M_I)\left(M_I - M_F + M_F MSd\right)T \\ &\Rightarrow t = T \end{aligned}$$

As a result, for the PM2 model, if $t = T$, the expected testing goal can be reached in any case.

3. Study on Parameters Relationships of PM2 Model

A range of growth curves are typically generated by planning models with diverse parameters. In this section, the impact of the single parameter on the growth curves of the PM2 model is examined. The data used in this section is derived from page 99 of the RGA7 Training Guide [31]. It is supposed a testing plan that has a short schedule and minimal test stages. The M_I is appointed as 217 miles and the goal is set as 350 miles, while the management strategy is assigned as 0.9 and the average fixed effectiveness factor is assigned to 0.60. Before discussing, the following assumptions are made: (i) the failures will be fixed instantly according to assigned fixed effectiveness factors once found during the test; (ii) corrective actions do not generate new failures.

3.1. Effect of the Testing Duration

Two parameters d and MS are set unchanged, while the effect of test duration variation on the growth curves is studied in this section. The original testing data are shown in Table 1.

Table 1. Planning data for testing duration.

Parameter	M_I/Miles	M_F/Miles	MS	d
Value	217	350	90%	0.60

Let the testing duration equal 5000 h, 10,000 h, and 15,000 h respectively, the planned growth curves are plotted as Figure 1 in terms of Equations (8) and (9). As shown in Figure 1, the dotted line shows the planned growth curve with T = 15,000 h. The solid line shows the planned growth curve with T = 10,000 h. The dashed line shows the planned growth curve with T = 5000 h. The dashed-dotted line shows the target of the developmental test. That is, as a subjective input parameter, the testing duration T will decide when the testings meet the goals and when the experiments end.

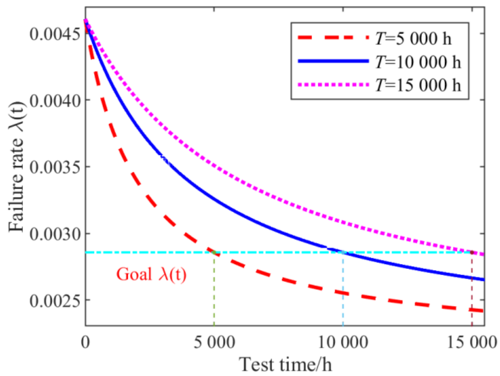

Figure 1. Effect of variant T on growth curves.

3.2. Effect of the Management Strategy

Two parameters T and d are fixed in this section, and the influence of different management strategies on the growth curves is studied. The original testing data are shown in Table 2. The testing duration is set to 10,000 h.

Let the management strategy equal 70%, 80%, and 90% respectively, the planned growth curves are plotted as Figure 2. As shown in Figure 2, the dotted line indicates the planned growth curve with MS = 90%. The solid line indicates the planned growth curve with MS = 80%. The dashed line indicates the planned growth curve

Table 2. Planning data for management strategy.

Parameter	M_I/Miles	M_F/Miles	T/h	d
Value	217	350	10,000	0.60

with *MS* = 70%. The dashed-dotted line indicates the goal of the developmental test. Similarly, the testing duration, *T*, still determines when it reaches the test goals and when the experiments end for any management strategy. It seems to be inconsistent with the definition and attributes of parameter *MS*. That is, the existing PM2 model cannot reflect well the variance of the management strategy parameter.

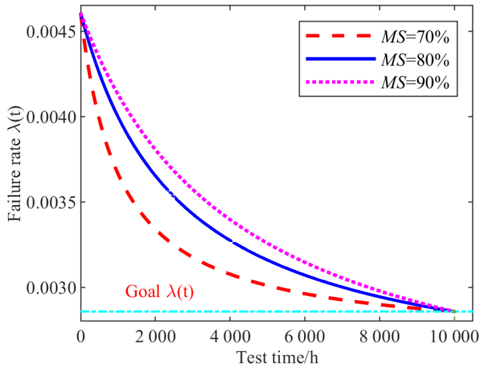

Figure 2. Effect of variant MS on growth curves.

3.3. Effect of the Fixed Effectiveness Factor

Two parameters *MS* and *T* are set unchanged in this section, and the effect of different fixed effectiveness factors on the growth curves is studied. The original testing data are shown in Table 3. Moreover, the testing duration is still set as 10,000 h, while the management strategy is set to be 0.9.

Table 3. Planning data for fixed effectiveness factor.

Parameter	M_I/Miles	M_F/Miles	T/h	MS
Value	217	350	10,000	0.90

Let parameter *d* equal 0.5, 0.6, and 0.7 respectively, according to Equations (8) and (9), the planned growth curves are plotted as Figure 3. The dotted line shows the planned growth curve with *d* = 0.7. The solid line indicates the planned growth curve with *d* = 0.6. The dashed line indicates the planned growth curve with *d* = 0.5. The dashed-dotted line still indicates the target of the developmental test. The testing duration still decides when the testing goals are met and when the tests end no matter how the fixed effectiveness factor changes. It seems to be inconsistent with the definition and attributes of parameter *d*. Thus, the existing PM2 model still cannot reflect the variance of the fixed effectiveness factor very well.

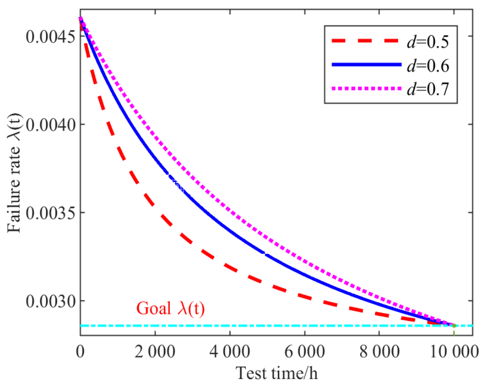

Figure 3. Effect of variant d on growth curves.

4. Discussion on the Relationship between *T* with *d* and *MS*

The PM2 model was used to discuss the lower boundaries of *MS, d*, and M_I in a specific case, and that planning curve can be significantly affected by the value of the product of parameters *d* and *MS* [27]. To reduce the failure risk of the reliability growth test, a discrimination inequality (Equation (10)) was proposed for the product of these two process management parameters. Following the sensitivity analysis of the products, it was observed that the product of these two parameters has a specific lower limit for all experimental inputs [29].

$$d \bullet MS > 1 - M_I / M_F \tag{10}$$

In terms of Equations (8) and (9), even if parameter T is a subjective input parameter, the planned experiment will end at time T and the target will be got accordingly, no matter how the fixed effectiveness factor and the management strategy change. In general, the product of MS and d will guide the actual reliability growth test, and that the product ($d \times MS$) should be compliant with the boundary condition, $(1 - M_I/M_F, 1]$. That is, the value of$MS$and$d$are bounded parameters. Otherwise, the final MTBF cannot reach M_F when the test ends.

According to Table 1, the product's boundary condition is (0.38, 1]. The dashed line in Figure 4 indicates that the curve becomes discontinuous and unintelligible when the product's value exceeds the boundary.

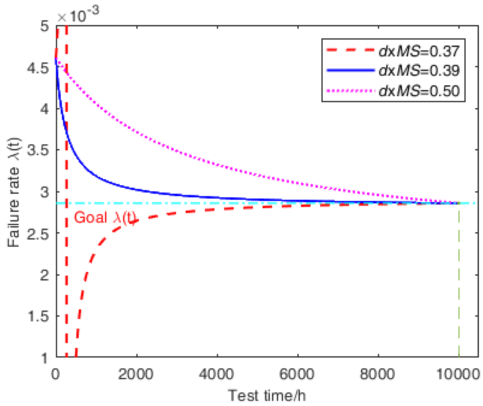

Figure 4. Effect of variant d×MS on growth curves.

Historical experimental data shows that the planned test duration (T) has a negative correlation with the fixed effectiveness factor (d) and the management strategy (MS). Thus, the testing duration (T) will be discussed as to how the product of these two parameters influences it. Ellner and Trapnell examined the fixed effectiveness factors of army systems using the reliability growth test and determined that the average is approximately 0.69 [19]. To fully comprehend the boundary of this parameter, the simulated test data is generated using a wider boundary condition of [0.40, 0.90]. Management strategy, which includes corrective ratio, is incorporated into the design. For systems that consist entirely of new or unproven technologies, it can be expected to be high, i.e., 0.95. For other systems using proven technologies, the chance to implement corrective actions is less, soMSmay be lower. Similarly, the management strategy will stay within 0.50 and 1.0 as the simulated data is generated. The execution time for the reliability growth test is usually between 2 months and 18 months. Therefore, the testing duration T is set from 1440 h to 12,960 h, as the boundary condition. A genetic algorithm was employed to expand the small sample data based on historical experimental data and the above boundary conditions. In result, 500 sets of data pairs were created, and the testing duration (T) has a negative impact on the product of MS and d. Such as, when $MS = 0.9504$ and $d = 0.8981$, the parameter T equal 1452 h. Similarly, when $MS = 0.5183$ and $d = 0.4179$, the parameter T equal 12,958 h.

The curve fitting module is chosen for data fitting and model solving in Matlab. Meanwhile, the fitting type is assigned as polynomial, and the degrees of d and MS are all assigned as five. The fitting results are shown in Table 4, in which SSE is the sum of squares due to error, R-square is the coefficient of determination, and RMSE is the root mean squared error.

Due to the higher input of test time T, the values of SSE and $RMSE$ are large, as demonstrated in Table 4. However, the adjusted R-square is 0.9986, very close to 1. Thus the fitting results are reasonable. Consequently,

Table 4. Result of fit goodness.

SSE	R-Square	Adjusted R-Square	*RMSE*
7.3×10^6	0.9987	0.9986	123.5

the solved nonlinear equation is recommended as Equation (11), in which the testing duration (*T*) is directly described by the management strategy (*MS*) and the fixed effectiveness factor (*d*).

$$\begin{aligned} T(d,MS) &= p_{00} + p_{10}d + p_{01}MS + p_{20}d^2 + p_{11}dMS + p_{02}MS^2 + p_{30}d^3 \\ &+ p_{21}d^2MS + p_{12}dMS^2 + p_{03}MS^3 + p_{40}d^4 + p_{31}d^3MS \\ &+ p_{22}d^2MS^2 + p_{13}dMS^3 + p_{04}MS^4 + p_{50}d^5 + p_{41}d^4MS \\ &+ p_{32}d^3MS^2 + p_{23}d^2MS^3 + p_{14}dMS^4 + p_{05}MS^5 \end{aligned} \tag{11}$$

where $p_{00}, p_{10}, p_{01}, p_{20}, p_{11}, p_{02}, p_{30}, p_{21}, p_{12}, p_{03}, p_{40}, p_{31}, p_{22}, p_{13}, p_{04}, p_{50}, p_{41}, p_{32}, p_{23}, p_{14}, p_{05}$ are all constant coefficients, the recommended value of these coefficients are shown in Table 5.

Table 5. Recommended coefficients with 95% confidence level.

Coefficient	Value	Confidence Bounds
p_{00}	1.311×10^5	$(6942, 2.552 \times 10^5)$
p_{10}	-8.233×10^5	$(-1.246 \times 10^6, -4.002 \times 10^5)$
p_{01}	-4.374×10^5	$(-1.084 \times 10^6, 2.089 \times 10^5)$
p_{20}	1.628×10^6	$(6.354 \times 10^5, 2.622 \times 10^6)$
p_{11}	2.82×10^6	$(1.716 \times 10^6, 3.925 \times 10^6)$
p_{02}	4.403×10^5	$(-1.08 \times 10^6, 1.961 \times 10^6)$
p_{30}	-1.302×10^6	$(-2.695 \times 10^6, 9.098 \times 10^4)$
p_{21}	-4.442×10^6	$(-5.684 \times 10^6, -3.201 \times 10^6)$
p_{12}	-3.042×10^6	$(-4.529 \times 10^6, -1.555 \times 10^6)$
p_{03}	1.506×10^4	$(-1.861 \times 10^6, 1.891 \times 10^6)$
p_{40}	3.745×10^5	$(-6.567 \times 10^5, 1.406 \times 10^6)$
p_{31}	2.723×10^6	$(1.898 \times 10^6, 3.548 \times 10^6)$
p_{22}	3.184×10^6	$(2.239 \times 10^6, 4.129e \times 10^6)$
p_{13}	1.294×10^6	$(2.812 \times 10^5, 2.306 \times 10^6)$
p_{04}	-2.219×10^5	$(-1.413 \times 10^6, 9.692 \times 10^5)$
p_{50}	-483	$(-3.138 \times 10^5, 3.129 \times 10^5)$
p_{41}	-5.811×10^5	$(-8.54 \times 10^5, -3.083 \times 10^5)$
p_{32}	-9.622×10^5	$(-1.256 \times 10^6, -6.679 \times 10^5)$
p_{23}	-7.128×10^5	$(-1.012 \times 10^6, -4.136 \times 10^5)$
p_{14}	-1.723×10^5	$(-4.675 \times 10^5, 1.229 \times 10^5)$
p_{05}	8.23×10^4	$(-2.264 \times 10^5, 3.91 \times 10^5)$

The fitting surface of the 500 data pairs is shown in Figure 5. The X-axis indicates parameter *d*, the Y-axis indicates parameter *MS*, the Z-axis indicates the proposed testing duration *T*, and that the unit of parameter *T* is h.

As a result, an extended reliability growth planning model is obtained, represented by Equation (12). This model takes into account the testing duration *T*, which is determined by the parameters *d* and *MS*. That is, the Equation (8) is replaced by Equations (11) and (12).

$$\lambda(t) = \lambda_A + (1-d)\lambda_B + d \cdot \frac{\lambda_B}{1 + \left(\dfrac{1}{T(d,MS)} \cdot \dfrac{1 - \dfrac{M_I}{M_F}}{d \cdot MS - \left(1 - \dfrac{M_I}{M_F}\right)} \right) \cdot t} \tag{12}$$

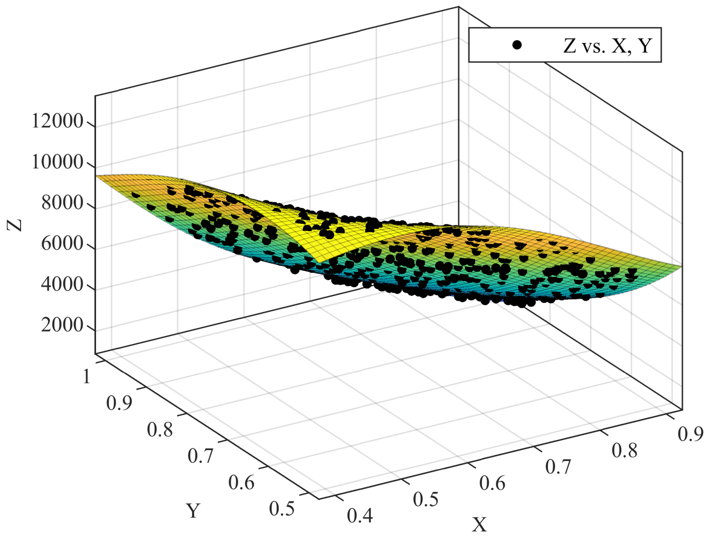

Figure 5. Fitting surface of 500 sets of data.

5. Illustrative Example

The data used in this study is derived from the manuscript of Crow [18]. Assuming that supplier ABC has conducted a development test for the network system. To ensure that the network system meets customer requirements, a minimum goal of 11.5 h for Mean Time Between Failures (MTBF) has been assigned. Based on the contract and a thorough analysis of similar projects, the *GPDM* is estimated to be 1.25. That is, the expected MTBF of the design margin is 25% larger than the requirement. The *MS* has been assigned a rating of 0.95 by the supplier experts due to its complexity and inclusion of an unproven technology system. The supplier also assumes a typical historical value of d equal to 0.70. Based on Equations (3) and (4), the value of M_I is calculated to be 4.8 h. Additionally, the duration of testing, $T(d, MS)$, is calculated to be 3069 h in terms of Equation (11). The planning curve generated by the proposed planning model is shown in Figure 6.

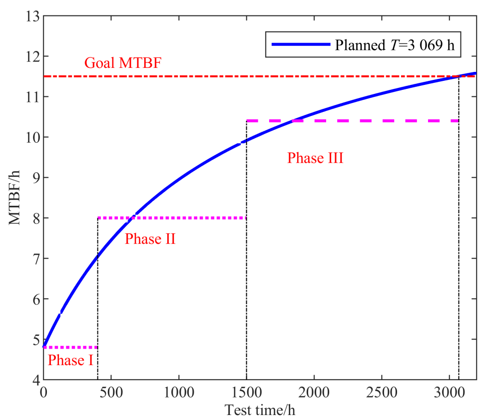

Figure 6. Comparison of the planning curve with the actual testing.

The initial reliability growth test, lasting 400 h, was carried out by Supplier ABC. During the phase I test, a total of 50 failures were detected and instantly resolved. The final MTBF of phase I, due to corrective actions implementation, is the same as the original MTBF of phase II: 8 h (400/50).

Consequently, the actual phased growth curves are plotted in Figure 6. The slide line indicates that any failures that arise are corrected instantly. The dotted lines and the dashed line show that failures are consistently addressed at the conclusion of each phase. That is, the values represented by the dotted lines and dashed line are the initial MTBF values of each stage, and also the MTBF values at the end of the previous stage. As shown in Figure 6, the system's reliability values experience a clear jump during each phase when the deferred corrective actions are implemented as a group. According to the proposed planning model, the reliability growth test will end at 3 069 h, which closely aligns with the actual testing duration in the Crow study that exceeded 2700 h [18]. In result, the planning curve matches well with the real three-phase growth curves. So the proposed extended model is reliable.

6. Case Study

A crankshaft handling robot was developed and tested in three stages at a specific plant. All corrective actions for observed B modes were performed at the end of the ongoing test phase. According to the customer's requirements, the goal for the Mean Time Between Failures (*MTBF*) of the handling system is set at a minimum of 200 h, with a known initial MTBF (M_I) of 95 h. The parameter d, has been assigned a value of 0.71 by factory experts. The results of the testing are displayed in Table 6.

Table 6. Failure data of crankshaft handling robot in developmental test.

Phase	Test Duration /h	No. of A Modes	No. of B Modes
I	570	1	5
II	1 050	1	7
III	2 580	2	13

Based on Table 6, the parameter$MS$is approximately calculated as (5 + 7 + 13)/(1 + 5 + 1 + 7 + 2 + 13) = 0.862. The testing duration $T(d, MS)$ can be calculated as 3900 h according to Equation (11). Consequently, the planning curves generated by the proposed planning model, are plotted in Figure 7.

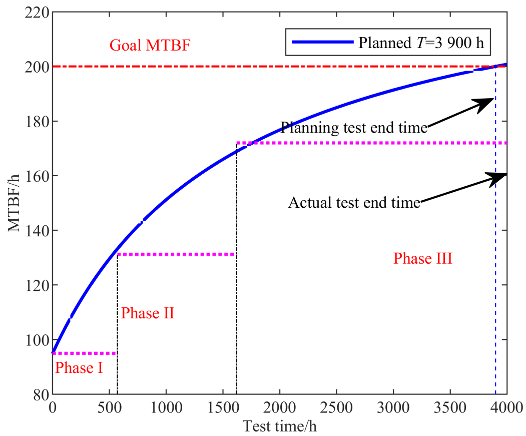

Figure 7. Planning performance in three testing phases.

Regarding the reliability growth curve, a total of 5 Type-B failures were observed during phase I. All necessary corrective actions will be implemented after phase I. Then, the next testing phase is currently underway and has revealed 8 failures, 7 of which are type-B failures. Therefore, the initial MTBF of phase II is estimated to be around 131.25 h. Similarly, the real MTBF of phase I is approximately 95 h, obtained by dividing 570 by 6, and the MTBF of phase III is found to be 172 h by dividing 2580 by 15.

In Figure 7, the dotted lines represent the actual step MTBF values. These lines display significant jumps between phases, indicating the simultaneous incorporation of corrective actions after each phase. The reliability growth curve, with instant failure correction, is illustrated by the solid line of the development test. As shown in Figure 7, the reliability growth test will reach the target and ends at 3900 h based on the proposed planning model. The practical test was terminated at 4000 h including three phases. The practical three-phase curves are closely matched by the planning curve as shown in Figure 7. That is, if the two management parameters d and MS are managed, the test duration can serve as an accurate measurement for a subjective value.

7. Conclusions

By considering the negative correlations between testing duration (T), effectiveness factor (d), and management strategy (*MS*) of the PM2 model, a nonlinear equation is introduced to describe the testing duration. This equation is then incorporated into an extended PM2 planning model. The following conclusions have been drawn from the study.

(1) As long as the experiment ends, the assigned test goal will be achieved with respect to the current PM2 model. The PM2 model doesn't show the changes in fixed effectiveness factors and management strategies well.

(2) During the test, it is important to strictly control the process management parameters *d* and *MS*. This will ensure that the required testing duration is accurate.

(3) The negative correlation between the planned testing duration with the fixed effectiveness factor and management strategy is discussed. Then a nonlinear equation of testing duration expressed by fixed effectiveness factor and management strategy is presented.

(4) An extended PM2 model is proposed, in which the testing duration (*T*) is expressed by fixed effectiveness factor (*d*) and management strategy (*MS*), so the parameter *T* is not a subjective input any longer.

(5) The proposed model is more reasonable and reliable since the subjective input is removed.

Further research should be conducted on the rationality of the nonlinear equation. In addition, this article is based on certain historical experimental data, a future research will focus on including more experimental data.

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The authors have no competing interests to declare that are relevant to the content of this article.

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